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SIR WM. R. HAMILTON, LL.D., President, in the Chair.

I. G. Abeltshauser, Esq. was elected a member of the Academy.

The special thanks of the Academy were voted to Miss Edgeworth for her donation of Wedgewood's Pyrometer.

Professor Mac Cullagh read the following communication.

On the Laws of Metallic Reflexion, and on the Mode of making Experiments upon Elliptic Polarization.

Several years ago, as the Academy are aware, I made an attempt to investigate the laws according to which light is reflected at the surfaces of metals, and I then proposed certain formulæ which represented, with sufficient accuracy, all the facts and experiments which I was able to collect upon the subject (see the Proceedings of the Academy, vol. i. p. 2, October, 1836; Transactions, vol. xviii. p. 71, note). But in order to test these formulæ satisfactorily, it was necessary to obtain measurements far more exact than any that had previously been made; and for this end I devised an instrument, which was constructed for me by Mr. Grubb, and of which a brief description has been given in the Proceedings, vol. i. p. 159. I regret to say, however, that no-

thing of much consequence has yet been done with the instrument. Some preliminary trials of its performance were indeed made in the summer of 1837, and the results of one of these shall presently be given; but an accidental strain which it suffered, while I was preparing to undertake a series of experiments, caused me to discontinue the observations at the time; and being then obliged to superintend the printing of my essay on the Laws of Crystalline Reflexion and Refraction (Transactions R. I. A., vol. xviii. p. 31), my attention was drawn afresh to this latter subject, respecting which some new questions suggested themselves, which I thought it right to discuss in notes appended to the essay. I was not afterwards at leisure to take up the experimental inquiry, until the beginning of the year 1839, when I began to think of putting the instrument in order for that purpose. The strain which it had suffered rendered some slight alterations necessary; and I now resolved to make additions to it also, with the view of operating upon the fixed lines of the spectrum, as a few trials had convinced me that measures sufficiently precise could not be obtained without employing light of definite refrangibility. I wished, moreover, to take the opportunity which the nature of the proposed experiments presented, of verifying the theory of Fresnel's rhomb, or rather of verifying, by means of the rhomb, the formulæ which Fresnel has given for computing the effects of total reflexion, when it takes place at the common surface of two ordinary media. I wrote therefore to Munich for several articles which I wanted; among others, for a set of rhombs cut at different angles, out of different kinds of glass. But while I was waiting for these some months elapsed, and in the meantime I got sight of a new theory, which, from its connexion with my former researches, possessed more immediate interest, and the pursuit of which, in conjunction with other studies and various engagements, caused me again to suspend the inquiry respecting the laws of metallic reflexion. I allude to the Dynamical Theory

of Crystalline Reflexion and Refraction, communicated to the Academy in December, 1839 (Proceedings, vol. i. p. 374). This was followed soon after by a general Theory of Total Reflexion (Proceedings, vol. ii. p. 96), founded on the same principles. The latter theory, forming a new department of physical optics, and involving the solution of questions not previously attempted, was *analytically* complete when it was communicated to the Academy in May, 1841; but its *geometrical* development has since required my attention from time to time, and has not yet been brought to that degree of simplicity of which it appears to be susceptible (see Proceedings, vol. ii. p. 174). Indeed I have found that, in this instance, the geometrical laws of the phenomena are by no means obvious interpretations of the equations resulting from the analytical solution of the problem; and in endeavouring to verify such supposed laws I have often been led to algebraical calculations of so complicated a nature that it has been impracticable to bring them to any conclusion, and I have been obliged, from mere weariness, to abandon them altogether. On returning, however, to the investigation, after perhaps a long interval of time, I have usually perceived some mode of eluding the calculations, or of directly deducing the geometrical law; and when the theory comes to be published in its final form, no trace of these difficulties will probably appear.

From the causes above-mentioned, combined with frequent absence from Dublin, the researches which I had entered upon, respecting the action of metals upon light, have been hitherto interrupted; and as it may still be some time before they are resumed, I venture, in the meanwhile, to submit to the Academy the results already spoken of, which were obtained on the first trial of the instrument, and which afford the best data that can yet be had for comparison with theory.

The results, it must be confessed, are those of very

rough experiments, made one evening (about the month of July, 1837) in company with Mr. Grubb, before I had received the instrument from his hands, and merely with the view of showing him, when it was finished, the kind of phenomena that I proposed to observe with it, and the mode of observing them. But the instrument was so far superior (in workmanship at least) to any apparatus previously employed for this sort of experiments, that it was impossible, without great negligence in using it, not to obtain measures of considerable accuracy. I did not, however, at the time, set much value on these measures, because I expected shortly to possess a series of observations made with every possible precaution; but having chanced to preserve the paper on which they were noted down, I was tempted, a few days ago, to try how far they agreed with my formulæ; and the agreement turns out to be so close, that I think myself justified in publishing them. Besides, it will be curious hereafter to compare them with more careful measurements.

Before we proceed, however, to the details of the experiments, it may be well to give the formulæ in a state fitted for immediate application. The light incident on the metal being polarized in a certain plane, let α denote the azimuth of this plane, or the angle which it makes with the plane of incidence; and as the reflected light will be elliptically polarized, or, in other words, will perform its vibrations in ellipses all similar and equal to each other, as well as similarly placed, put θ for the angle which either axis of any one of these ellipses makes with the plane of incidence, and let β be another angle, such that its tangent may represent the ratio of one axis of the ellipse to the other. Then when the optical constants M and χ (of which I suppose the first to be a number greater than unity, and the second an angle less than 90°) are known for the particular metal, the angles θ and β may be computed for any value of α, at any given angle of incidence, by the following formulæ:

$$\left.\begin{aligned}\tan 2\theta &= \frac{(\nu'-\nu)\sin 2a}{2f+(\nu'+\nu)\cos 2a},\\ \sin 2\beta &= \frac{2g\sin 2a}{\nu'+\nu+2f\cos 2a};\end{aligned}\right\}\qquad\text{(A)}$$

in which f and g are constant quantities given by the expressions

$$f=\left(\mathrm{M}-\frac{1}{\mathrm{M}}\right)\cos\chi,\qquad g=\left(\mathrm{M}+\frac{1}{\mathrm{M}}\right)\sin\chi,\qquad\text{(B)}$$

and ν, ν' are quantities depending on the angle of incidence i, in the following way. Let i' be an angle such that

$$\frac{\sin i}{\sin i'}=\frac{\mathrm{M}}{\cos\chi},\qquad\text{(C)}$$

and put

$$\frac{\cos i}{\cos i'}=\mu,\qquad\text{(D)}$$

then will

$$\nu=\frac{1}{\mu}-\mu,\qquad \nu'=\frac{f^2+g^2}{\nu}.\qquad\text{(E)}$$

The angles θ and β are given by immediate observation with the instrument; and from their values at any incidence, and for any azimuth a of the plane of primitive polarization, we can find the constants M and χ, which we may afterwards use to calculate the values of θ and β for all other incidences and azimuths, in order to compare them with the observed values. It is indifferent, in the formulæ, whether θ be referred to the major or the minor axis of the elliptic vibration, as also whether $\tan\beta$ be the ratio of the minor to the major axis, or the reciprocal of that ratio; but in what follows we shall suppose θ to be the inclination of the plane of incidence to that axis, which, when a is 45° or less, is always the major axis; and β shall be supposed less than 45°, in order that its tangent may represent the ratio of the minor axis to the major.

When the azimuth a is equal to 45°, the formulæ (A) become

$$\tan 2\theta=\frac{\nu'-\nu}{2f},\qquad \sin 2\beta=\frac{2g}{\nu'+\nu};\qquad\text{(F)}$$

from which we may deduce the remarkable relation

$$\frac{\tan 2\beta}{\cos 2\theta}=\frac{g}{f},\qquad\text{(G)}$$

showing that, in the case supposed, the ratio of $\tan 2\beta$ to $\cos 2\theta$ is independent of the angle of incidence. In the experiments which I made with Mr. Grubb this azimuth was always 45°; and the following Table contains the results of observation compared with those obtained by calculation from formulæ (F). The experiments were made upon a small disk of speculum metal; and in the calculations I have taken $M = 2.94$, $\chi = 64° 25'$.

Angle of Incidence.	Value of θ.		Value of β.	
	Observed.	Calculated.	Observed.	Calculated.
65°	27° 55′	27° 53′	28° 0′	28° 0′
70	15 41	15 44	33 7	33 1
75	− 8 45	− 9 16	34 10	34 6
80	−30 15	−29 25	27 0	26 53
84	−37 22	−37 25	16 47	17 17

The light used in these observations was that of a candle placed at a short distance, and was admitted through small apertures at the ends of the tubes. (See the description of the instrument in the Proceedings, vol. i. p. 159). The Nicol's prism in the first tube having been secured in a position in which its principal plane was inclined 45° to the plane of incidence, and the two arms having been set at the proper angle with the surface of the metal, the Fresnel's rhomb and the Nicol's prism in the second tube were moved simultaneously, until the image of the candle became as faint as possible. Had light perfectly homogeneous been employed, the image could have been made to vanish altogether; but instead of vanishing, it became highly coloured, and our rule in observing was to make the blue at one side of it, and the red at the other, equally vivid, so as to get results which should belong, as nearly as possible, to the mean ray of the spectrum. When this was done, the angles θ and β (subject however to certain corrections which will be hereafter explained) were respectively read off from the divided circles belonging to the rhomb and the prism. The observations were made at large

incidences, because it is within the last thirty degrees of incidence that the phenomena go through their most rapid changes.

If we now cast our eyes on the above table, making due allowance for the uncertainty arising from the *dispersion* of the metal, we shall be struck with the agreement between the calculated and observed numbers. The differences are greatest in the last two observations, which however were really the first; for the observations were made in the inverse order of the incidences, and their accuracy may have improved as they went on. However that may be, the differences are quite within the limits of the errors of observation; and they are actually less than those which Fresnel found to exist between calculation and experiment in the much simpler case of reflexion at the surface of a transparent ordinary medium, when he proceeded to verify the formula which he had discovered for computing the effect of such reflexion.—See the Table which he has given in the *Annales de Chimie,* tom. xvii. p. 314.

It may seem extraordinary that these experiments should have been in my possession for nearly six years, before I became aware of their close agreement with my formulæ; but the fact is, that I did not regard them with much interest, because, from the circumstances in which they were made, I did not expect more than a general accordance with theory. And even now, I am in no haste to infer the absolute exactitude of the formulæ, though they are found to represent the phenomena so well. It was far more allowable to infer that the formula of Fresnel was exact in the case just mentioned, though it appeared to represent the phenomena less perfectly. For, to say nothing of the small number of our experiments, the present is a much more complicated case, and the phenomena depend on two constants instead of one, so that the formulæ might be slightly altered, and yet perhaps continue to agree very well with rough experiments. Where there is only one constant this is not so probable. Again,

there is one of the quantities in the preceding formulæ which may be greatly altered without producing more than a slight effect on the values of θ and β. This quantity is the ratio of $\sin i$ to $\sin i'$, which, according to the value in formula (C), is a number so large as to make the angle i' always small, so that its cosine never differs much from unity; and therefore if the above ratio were taken equal to any other large number, the value of μ in formula (D) would remain nearly the same, and consequently the values of θ and β would be but slightly changed.

It is with regard to the value of μ as a function of the incidence that I entertain the greatest doubts, and if any defect shall be found in the formulæ I think it will be here. The relations (C) and (D), from which μ may be deduced in terms of i, were not indeed adopted without strong reasons; but I am not entirely satisfied with them, because, when we reverse the problem, and seek to determine the constants M and χ from the observed values of θ and β at a given incidence, the results are rather complicated and involved, though the approximate determination is easy enough. As the formulæ are in a great measure built upon conjecture, we must not be disposed to receive them without the strongest experimental proofs; and it will certainly require experiments of no ordinary accuracy to decide some of the questions which may be raised respecting them.

When plane-polarized light is incident on a metal, if its vibrations be resolved in directions parallel and perpendicular to the plane of incidence, the effect of the reflexion is to change unequally the phases of the resolved vibrations; and it may be useful to have the formulæ which express the difference of phase after reflexion, and the ratio of the amplitudes of vibration. Put ϕ for the difference of phase; and supposing, for simplicity, the incident light to be polarized in an azimuth of 45°, let σ be angle less than 45°, such that $\tan\sigma$ may represent the ratio of the reflected amplitudes

respectively perpendicular and parallel to the plane of incidence; then we shall have

$$\tan\phi = \frac{2g}{\nu' - \nu}, \qquad \cos 2\sigma = \frac{2f}{\nu' + \nu}; \tag{H}$$

from which we may infer that

$$\sin\phi \tan 2\sigma = \frac{g}{f}, \tag{I}$$

or that the product on the left side of the last equation is independent of the angle of incidence. It is to be observed that the relations (G) and (I) are independent of the value of μ, and may hold good though that value should require to be changed.

All the preceding formulæ are merely mathematical consequences of those which I published long ago in the Transactions of the Academy (vol. xviii. p. 71). The formulæ which I had previously given in the Proceedings (vol. i. p. 2) are slightly different, and, I think, less likely to be exact, because they are less simple, and do not lead to any of the remarkable relations which may be deduced from the others.

Having had occasion, in the course of the few experiments which I made with the instrument before mentioned, to study the nature of Fresnel's rhomb, which constitutes an important part of it, I shall here describe the method which must be followed in order to obtain true results, when the rhomb is employed in observations on light elliptically polarized. A ray in which the vibrations are supposed to be elliptical is given, and what we want is to determine the ratio of the axes of the elliptic vibration, and their directions with respect to a fixed plane passing through the ray; in other words, to determine the angles which we have denoted by β and θ in the case of a ray reflected from a metal. For this purpose the ray is admitted perpendicularly to the surface at one end of the rhomb, and after having suffered two total reflexions within, passes out perpendicularly to the sur-

face at the other end. Then causing the rhomb to revolve about the ray, we shall find two positions of it in which the emergent light will be plane-polarized, these positions being readily indicated by a Nicol's prism interposed between the rhomb and the eye; for such a prism, by being turned round the ray, can make the light totally disappear when it is plane-polarized, but not otherwise. These two positions of the rhomb will be exactly 90° from each other; in one of them the principal plane of the rhomb (the plane of reflexion within it) will be parallel to the major axis of the elliptic vibration, and the angle which it makes with the plane of incidence on the metal will be equal to θ: while in the same position the angle which the principal plane makes with the plane of polarization of the emergent ray (as given by the Nicol's prism) will be equal to β. In the other position, the principal plane will be parallel to the minor axis of the elliptic vibration, and the corresponding angles will be equal to $90°-\theta$ and $90°-\beta$ respectively. This, however, proceeds on the supposition that the rhomb is exact. When it is not so, which is of course the proper supposition, and a very necessary one in the experiments with which we are concerned, there will still be, generally speaking, two positions of it in which the emergent ray will be plane-polarized, or in which a disappearance of the light may be produced by the Nicol's prism; but these positions will no longer be 90° from each other, nor will the principal plane, in either of them, coincide with an axis of the elliptic vibration. If we now measure the angles between the different planes as before, and denote them by θ', β' in the first position, and by $90°-\theta''$, $90°-\beta''$ in the second, we shall find that θ' and θ'' are unequal, but we shall have β' equal to β''. The values of θ and β will then be given by the formulæ

$$\theta = \frac{\theta' + \theta''}{2}, \qquad \cos 2\beta = \frac{\cos 2\beta'}{\cos(\theta' - \theta'')}. \qquad \text{(K)}$$

The error of the rhomb may easily be found. Supposing

the vibrations to be resolved in directions parallel and perpendicular to its principal plane, the rhomb is intended to produce a difference of 90° between the phases of the resolved vibrations, or to alter by that amount the difference of phase which may already exist; but the effect really produced is usually different from 90°, and this difference, which I call ε, is the error of the rhomb. The value of ε is given by the formula

$$\tan \varepsilon = \frac{\sin(\theta' - \theta'')}{\tan 2\beta}; \tag{L}$$

and as the error of the rhomb is a constant quantity, we have thus an equation of condition which must always subsist between the angles $\theta' - \theta''$ and β. For any given rhomb the sine of the first of these angles is proportional to the tangent of twice the second, and therefore $\theta' - \theta''$ constantly increases as β increases towards 45°, that is, as the axes of the elliptic vibration approach to equality. When β is equal to $45° - \frac{1}{2}\varepsilon$, we have $\theta' - \theta'' = 90°$; and for values of β still nearer to 45°, the value of $\sin(\theta' - \theta'')$ becomes greater than unity, that is to say, it becomes impossible, by means of the rhomb, to reduce the light to the state of plane-polarization. This is a case that may easily happen with an ordinary rhomb in making experiments on the light reflected from metals; because at a certain incidence, and for a certain azimuth of the plane of primitive polarization, the reflected light will be circularly polarized.

The rhomb which I used in the experiments tabulated above, was made by Mr. Dollond, and was perhaps as accurate as rhombs usually are; it was cut at an angle of $54\frac{1}{2}°$, as prescribed by Fresnel. Its error was about 3°, and the value of $\theta' - \theta''$, at the incidence of 75°, was about 7°. But in another rhomb, also procured from Mr. Dollond, and cut at the same angle, the value of $\theta' - \theta''$, under the same circumstances, was about 20°, and the value of ε was therefore

about 8°. The angle given by Fresnel was calculated for glass of which the refractive index is 1.51; and the errors of the rhombs are to be attributed to differences in the refractive powers of the glass. I was not at all prepared to expect errors so large as these when I began to work with the rhomb, and they perplexed me a good deal at first, until I found the means of taking them into account, and of making the rhomb itself serve to measure and to eliminate them. The value of the rhomb as an instrument of research is much increased by the circumstance that it can thus determine its own effect, and that it is not at all necessary to adapt its angle exactly to the refractive index of the glass. It may also be remarked, that this circumstance affords a method of directly and accurately testing the truth of the formulæ which Fresnel has given for the case of total reflexion at the separating surface of two ordinary media; for we have only to measure the angle of the rhomb and the refractive index of the glass, and to compute, by Fresnel's formula, the alteration which the rhomb ought to produce in the difference between the phases of the resolved vibrations; which alteration of phase we may then compare with that deduced, by means of the formulæ (K) and (L), from direct experiment.

If, in each position of the rhomb, we measure the angle which the plane of polarization of the emergent ray makes with the plane of incidence on the metal, and call the two angles respectively γ', γ'', we shall have

$$\gamma' = \theta' - \beta', \qquad \gamma'' = \theta'' + \beta', \tag{M}$$

and therefore

$$\gamma' + \gamma'' = \theta' + \theta'' = 2\theta, \qquad 2\beta' = \gamma'' - \gamma' + \theta' - \theta''; \tag{N}$$

from which it appears that if the rhomb were perfectly exact, that is, if θ' and θ'' were equal to each other, the angle θ would be half the sum of γ', γ'', and the angle β half their difference. It would then be sufficient to measure the angles γ' and γ'', in order to get θ and β accurately. And if the

rhomb were erroneous, the true value of θ would still be half the sum of γ', γ''; but the true value of β would not be discoverable without measuring the angles θ', θ'', by the help of which it can be deduced from the second of formulæ (N), combined with the second of formulæ (K). Nor can we discover whether the rhomb is erroneous or not, without measuring the angles θ', θ''; and therefore as these angles *must* be measured in any case, the former method of determining θ and β is to be preferred.

In making experiments on elliptically polarized light, a plate of mica or any other doubly refracting crystal, placed perpendicular to the ray, may be used instead of Fresnel's rhomb. If the thickness of the crystalline plate be such that the interval between the two rays which emerge from it is equal to the fourth part of the length of a wave, for light of a given refrangibility, the plate will, for such light, perform all the functions of the rhomb; the principal plane of the rhomb being represented by the plane of polarization of one of the emergent rays. But unless the light be perfectly homogeneous, this method is liable to great inaccuracy in practice, since the effect of the plate in producing or altering the difference of phase between the two rays which interfere on their emergence from it, is inversely proportional to the length of a wave, and will therefore be extremely different for light of different colours, and will change very perceptibly even within the limits of the same colour. It is true, the effect of the rhomb also varies with the colour of the light: but this variation is trifling compared with that which exists in the other case. It was for this reason that I employed the rhomb in my experiments, instead of a crystalline plate. The apparatus, however, is much simplified by using such a plate; and if any one chooses to do so, and to work with homogeneous light, he must take care to follow, in every respect, the directions which I have given for conducting experiments with the rhomb. The two cases are precisely

similar; and if it be necessary not to neglect the errors of the rhomb, it is certainly not less necessary to take into account those which may arise from a want of accuracy in the thickness of the plate, considering how difficult it is to make the thickness correspond exactly to the particular ray which we wish to observe.

I have been induced to enter into these particulars, respecting the mode of making experiments on elliptic polarization, because the subject is one which has not hitherto been studied; nor does it seem to have occurred to any one that any precaution was requisite beyond that of getting the rhomb cut as nearly as possible at the proper angle, or the crystalline plate made as nearly as possible of the proper thickness. This, indeed, was quite sufficient for ordinary purposes. For example, light polarized in a plane inclined 45° to the principal plane of the rhomb or of the plate, would, as far as the eye could judge, be circularly polarized after passing through either of them. Notwithstanding a certain error in the angle of the one, or in the thickness of the other, such light would, when analysed by a rhomboid of Iceland-spar, give two images always sensibly equal in intensity. But an error which could not be at all detected in this way, might produce a very great effect in such experiments as those upon the metals, and, for the purpose of comparison with theory, might render them entirely useless, if in the first method of observing we relied upon one set of observations, taking (suppose) the values of θ' and β' for the true values of θ and β; or if, in the second method, we contented ourselves with merely measuring the angles γ' and γ''.

The necessity of attending to the foregoing rules and remarks will appear from an examination of the experiments of M. de Senarmont, published in the *Annales de Chimie*, tom. lxxiii. pp. 351–358. In these very elaborate experiments, which were made upon light reflected at various incidences from steel and speculum metal, the author followed

a plan similar to that which I have adopted, and which, in a general way, I had previously sketched in the Proceedings of the Academy (vol. i. p. 159). There was this difference, however, that he used a plate of mica instead of Fresnel's rhomb. Now as he worked with common white light, the use of the mica plate must have rendered two kinds of errors unavoidable. In the first place, it would be impossible always to take the observations for the same ray of the spectrum; and next, as a consequence of this, the thickness of the plate would be generally inexact for the particular ray to which the observations happened to correspond. If the thickness of the plate were exact for a certain ray, it would be very sensibly inexact even for the neighbouring parts of the spectrum; and as the part of the spectrum to which the observations belonged was continually changing, the results obtained for different incidences and azimuths would not be comparable with each other, even though, in each separate case, the error of the plate were allowed for and eliminated. The values of θ, however, as determined by M. de Senarmont, would be correct, so far as *this* error is concerned; those of β alone would be erroneous. For the values of θ were determined in two ways: by measuring the angles θ', θ'', and taking their sum for 2θ; also by measuring the angles γ', γ'', and taking their sum for the same quantity. Now each of these methods gives a true value of θ, because by the preceding formulæ we have $2\theta = \theta' + \theta'' = \gamma' + \gamma''$; and this accounts for the agreement, shown by the tables of M. de Senarmont, between the values* of 2θ obtained by these different methods. But the values of β were deduced from the angles γ', γ'', by simply making their dif-

* Or rather the values of $180^\circ + 2\theta$; because the angle ω, the double of which appears in the tables of M. de Senarmont, is equal to $90^\circ + \theta$. The angles which he calls γ_1 and γ_2 are equal to $90^\circ + \gamma''$ and $90^\circ + \gamma'$ respectively. It therefore comes to the same thing, whether the one set of angles or the other is supposed to be measured. The letter β has the same signification in both notations.

ference equal to 2β; and we see by the second of formulæ (N) that, when the plate is not of the proper thickness, this value of 2β is erroneous by the whole amount of the angle $\theta' - \theta''$, the difference between β' and β being supposed so small that it may be neglected. As M. de Senarmont proceeded on the common assumption that when the thickness of the plate has been adjusted to that part of the spectrum to which the observations are intended to refer, it may afterwards, through the whole series of experiments, be regarded as exact, he necessarily conceived θ' and θ'' to be the same angle; and it was on the principle of taking an average between two measures of the same quantity, that he made the supposition $2\theta = \theta' + \theta''$, which happened to be correct. When therefore he found θ' and θ'' to be different, he of course looked upon the difference as merely an error of observation, which it would be superfluous to tabulate. Not having the values of this difference, therefore, we have not the means of immediately correcting the values of 2β. But as observations were made for several azimuths at each angle of incidence, we may use the values of θ to determine those of β; for when at any incidence (except that of maximum polarization, where $\theta = 0$ for all azimuths) the values of θ are known for two given values of a, we can deduce the corresponding values of β, without any other theory than that of the composition of vibrations. The values of β so deduced must indeed be expected to be very inaccurate, partly because of errors in the observed values of θ, partly because the observations in different azimuths do not answer to the same ray of the spectrum; but they will be accurate enough to show the great amount of the error committed by neglecting the difference $\theta' - \theta''$. For example, putting θ_0 and β_0 for the values of θ and β when $a = 45°$, M. de Senarmont gives, at the incidence of 60° upon steel, $2\theta_0 = 64° 15'$ (taking the mean of his two determinations), and for the azimuths 55°, 30°, 25°, he gives 2θ equal to 88° 5′, 37° 2′, and 29° 36′ respectively. Combining these

values of 2θ in succession with that of $2\theta_0$, we get for $2\beta_0$ the series of values 32° 38′, 33° 28′, 34° 30′; the differences between which are to be attributed to the causes above stated. The mean value of $2\beta_0$ thus found is 33° 32′; while its value, as given by M. de Senarmont, is only 28° 41′. The difference 4° 51′ is the value of $\theta' - \theta''$, which, divided by the tangent of $2\beta_0$, gives 7° 19′ for the mean value of ϵ, the error of the mica-plate corresponding to that part of the spectrum which was observed at the incidence of 60°.

At incidences nearer the angle of maximum polarization, the errors are probably much greater. Beyond that angle they again diminish, and in some cases they almost vanish. Thus, at the incidence of 85° upon steel, with the value of $2\theta_0$ and the value of 2θ corresponding to $a = 20°$, we get, by computation, a value of $2\beta_0$ which differs only by a few minutes from that given by M. de Senarmont. Nearly the same thing happens at the same incidence when we take $a = 25°$. In these cases therefore the results belong to that particular ray for which the thickness of the plate was exact.

The observations of M. de Senarmont on speculum metal were not carried beyond the incidence of 60°. He states that he was unable to observe at higher incidences, on account of the uncertainty arising from the *dispersion* of the metal; but though this cause operated in some degree, his embarrassment must have been really occasioned by the increasing magnitude of the difference $\theta' - \theta''$, as he approached the angle of maximum polarization; that difference being perhaps twice as great as in the case of steel. My own experiments on speculum metal were all made, as has been seen, at incidences *greater* than 60°.

The experiments of M. de Senarmont do not at all agree with the formulæ; and therefore I have been obliged to analyse his method of observation, and to show that it could not lead to correct results. It is to be regretted that his

method was defective, as the zeal and assiduity which he has displayed in the inquiry would otherwise have put us in possession of a large collection of valuable data.

I shall conclude by saying a few words respecting the intensity of the light reflected by metals. The formulæ for computing this intensity have been given in the Transactions of the Academy, in the place already referred to; but they may be here stated in a form better suited for calculation. If we suppose ψ and ψ' to be two angles, such that

$$\cot \psi = \frac{\text{M}}{\mu}, \qquad \cot \psi' = \text{M}\mu, \tag{O}$$

and then take two other angles ω, ω', such that

$$\cos\omega = \sin 2\psi \cos\chi, \qquad \cos\omega' = \sin 2\psi' \cos\chi, \tag{P}$$

we shall have

$$\tau = \tan \tfrac{1}{2}\,\omega, \qquad \tau' = \tan \tfrac{1}{2}\,\omega', \tag{Q}$$

where τ is the amplitude of the reflected rectilinear vibration, when the incident light is polarized in the plane of incidence, and τ' is the amplitude of the reflected vibration when the incident light is polarized perpendicularly to that plane; the amplitude of the incident vibration being in each case supposed to be unity. Hence when common light is incident, if its intensity be taken for unity, the intensity I of the reflected light will be given by the formula

$$\text{I} = \tfrac{1}{2}(\tan^2 \tfrac{1}{2}\,\omega + \tan^2 \tfrac{1}{2}\,\omega'). \tag{R}$$

If with the values of M and χ determined by my experiments we compute, by the last formula, the intensity of reflexion for speculum metal at a perpendicular incidence, in which case $\mu = 1$, we shall find $\text{I} = .583$. This is considerably lower than the estimate of Sir William Herschel, who, in the Philosophical Transactions for 1800 (p. 65), gives .673 as the measure of the reflective power of his specula. The same number, very nearly, results from taking the mean of Mr. Potter's observations (Edinburgh Journal of Science, New Series, vol. iii. p. 280). It might seem therefore that

the formula is in fault; but I am inclined to think that the metal which I employed had really a low reflective power. Its angle of maximum polarization was certainly much less than that of the speculum metal used by Sir David Brewster (Phil. Trans. 1830, p. 324), who states the angle to be 76°, whereas in my experiments it was only about $73\frac{1}{2}°$; and any increase in this angle, by increasing the value of M, raises the reflective power. On the other hand, the maximum value of β (when $\alpha = 45°$) was greater than that given by Sir David Brewster, namely, 32°; and any increase in β tends also to increase the reflective power. Now it is not unreasonable to suppose that the highest values of both angles may be most nearly those which belong to the best specula; and accordingly if we take 76° for the incidence of maximum polarization, and retain the maximum value of β, namely 34° 37′, which results from my experiments, we shall get $M = 3.68$, $\chi = 66° 16'$, and the value of I at the perpendicular incidence will come out equal to .662, which scarcely differs from the number given by Herschel.

It is clear from what precedes that the optical constants are different for different specimens of speculum metal, and this is no more than we should expect, from the circumstance that the metal is a compound, and therefore liable to vary in its optical properties from variations in the proportion of its constituents; but I am disposed to believe that the same thing is generally true, though of course in a less degree, of the simple metals, so that in order to render the comparison satisfactory, the measures of intensity should always be made on the same specimen which has furnished the values of M and χ. There is one metal, however, with respect to which there can be no doubt that the experiments of different observers are strictly comparable, when it is pure, and at ordinary temperatures; I mean mercury. For this metal Sir David Brewster states the angle of maximum pola-

rization to be 78° 27′, and the maximum value of β, when $a = 45°$, to be 35°; from which I find $\text{M} = 4.616$, $\chi = 68°\ 13'$, and, at the perpendicular incidence, $\text{I} = .734$. Now Bouguer observed the quantity of light reflected by mercury, but not at a perpendicular incidence. His measures were taken at the incidences of 69° and $78\frac{1}{2}°$, for the first of which he gives, by two different observations, .637 and .666; for the second, by two observations, .754 and .703, as the intensity of reflexion. (See his *Traité d'Optique sur la Gradation de la Lumière*, Paris, 1760; pp. 124, 126). If we make the computation from the formula, with the above values of M and χ, we find the quantities of light reflected at these two incidences to be, as nearly as possible, equal to each other, and to seven-tenths of the incident light, the intensity of reflexion being a minimum at an intermediate incidence; and if we suppose these quantities to be really equal at the incidences observed by Bouguer, we must take the mean of all his numbers, which is .69, as the most probable result of observation. This result differs but little from one of the two numbers given by him at each incidence, and scarcely at all from the result of calculation.

The angle at which the intensity of reflexion is a minimum, when common light is incident, may be found from the formula

$$\left(\text{M} + \frac{1}{\text{M}}\right)\left(\mu + \frac{1}{\mu}\right) = \left(\text{M} - \frac{1}{\text{M}}\right)\sqrt{(f^2 + g^2)} - 4\cos\chi, \qquad (\text{s})$$

which gives the value of μ, and thence that of i. This incidence for mercury is, by calculation, 75° 15′, and the minimum value of I is .693, which is less than its value at a perpendicular incidence by about one-eighteenth of the latter. According to the formulæ, the reflexion is always total at an incidence of 90°.

Rev. Charles Graves communicated certain extracts from

a work of the late Dr. Cheyne, on a Deranged State of the Faculty of communicating by Speech or Writing.*

Dr. Allman read a paper "on a New Genus of Hydra-form Zoophytes."

The author discovered the animal on which he founded the new genus in the Grand Canal near Dublin, in October, 1842. The genus of which this zoophyte constitutes as yet the only known species, will find a place in the family of the *tubulariadæ*, and occupies a position between *coryne* and *tubularia*, differing from the former in the possession of a polypedome, and from the latter in the scattered arrangement of its tentacula. The tentacula, as in both the last mentioned genera, are filiform; and in this character a point of distinction is at once found between the new genus and *Hermia*, Johnst.

To the new zoophyte Dr. Allman assigned the name *Cordylophora lacustris*.

May 22.

SIR WM. R. HAMILTON, LL.D., President, in the Chair.

Right Hon. the Earl of Dunraven was elected a member of the Academy.

Dr. Osborne read some observations on the deprivation of the faculty of speech while the intellect remains entire, and in which the defect does not arise from paralysis of the vocal organs. The communication was intended as a sequel

* This work having been since published, the extracts are not here given.